

# **Superresolution Optical Fluctuation Imaging (SOFI) using quantum dots**

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Dertinger et al., PNAS 2009; Optics Express 2010; Angew Chemie 2010

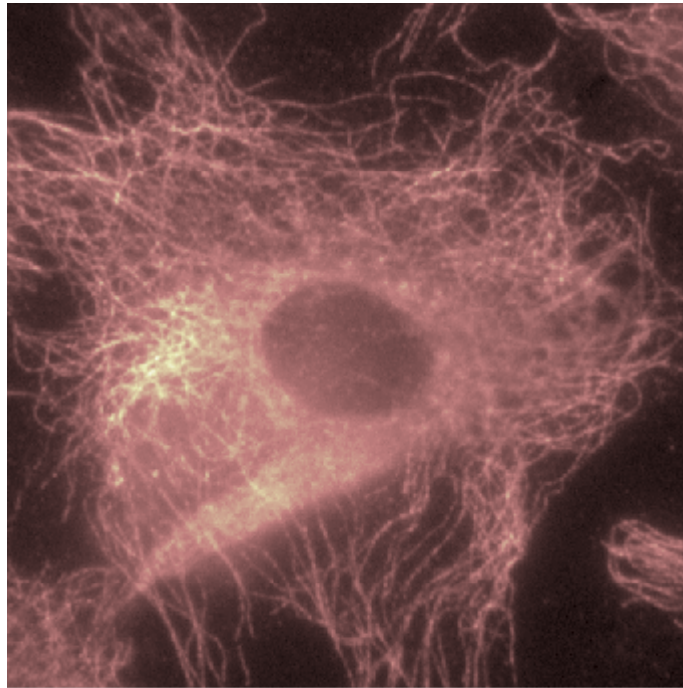
# Superresolution Optical Fluctuation Imaging (SOFI)

1. The fluorescent label has to exhibit at least two different emission states. For example, these states can be a fluorescent and a non-fluorescent one, but in principle any two or more states which are optically distinguishable will do.
2. Different emitters have to switch between states repeatedly and independently from each other.

**Thomas Dertinger, Ryan Colyer, Joerg Enderline**

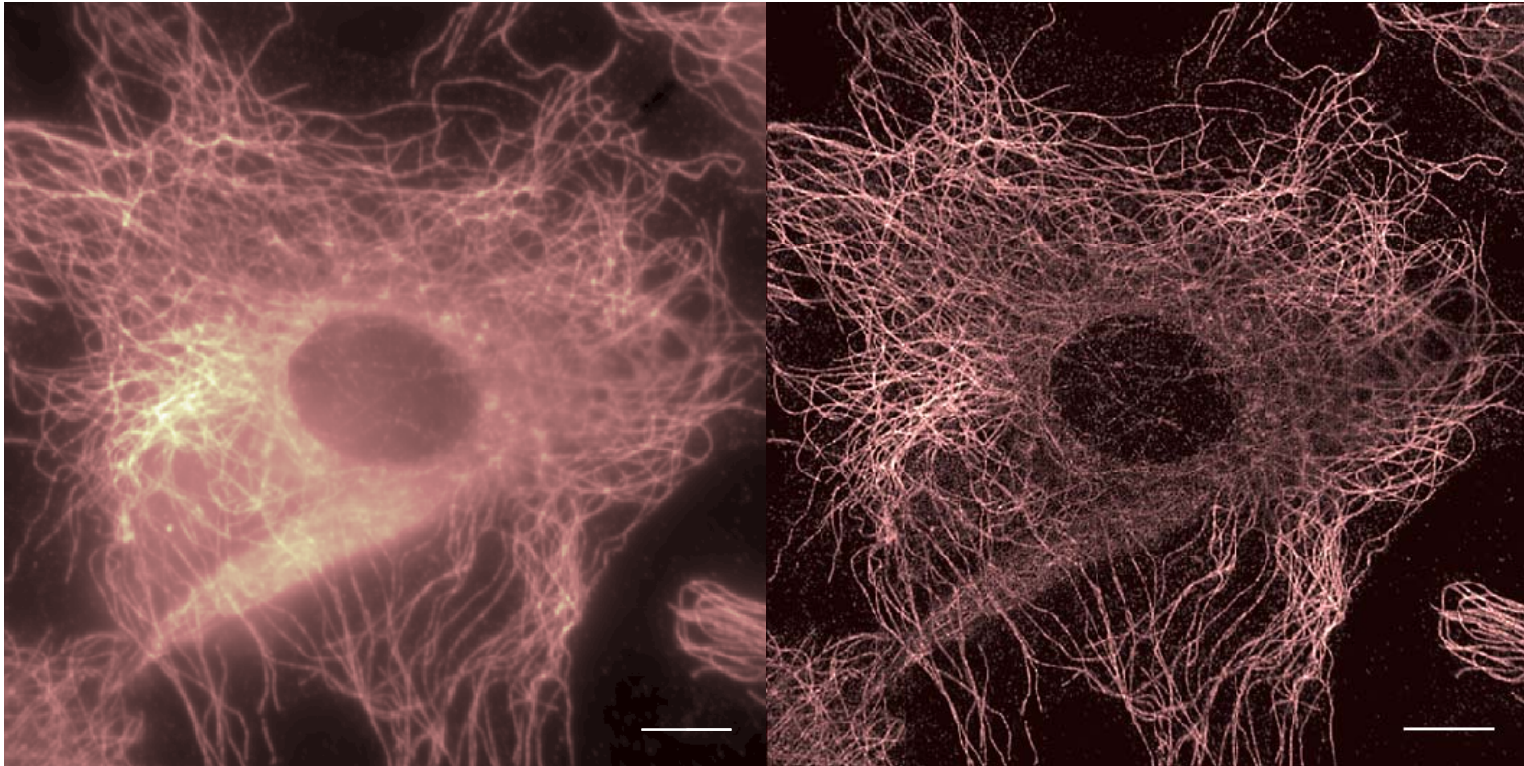
# SOFI

Acquire movie on a fluctuating signal (any probe fluctuation)



3T3 fibroblast; quantum dot labeled tubulin network

## SOFI features Super-resolution in 3D



Resolution enhancement: 2x in 3D

Scalebar: 10  $\mu\text{m}$

# image formation

$$F(\mathbf{r}, t) = \sum_{k=1}^{\text{all molecules}} U(\mathbf{r} - \mathbf{r}_k) \varepsilon_k f_k(t)$$

image on CCD

point spread function

brightness of  $k^{\text{th}}$  molecule

time-dependent brightness fluctuations of  $k^{\text{th}}$  molecule

**PSF inside the sum over emitters**

# Correlation Analysis

To do SOFI, we correlate the fluctuations:  $\delta F(t) = F(t) - \langle F(t) \rangle_t$

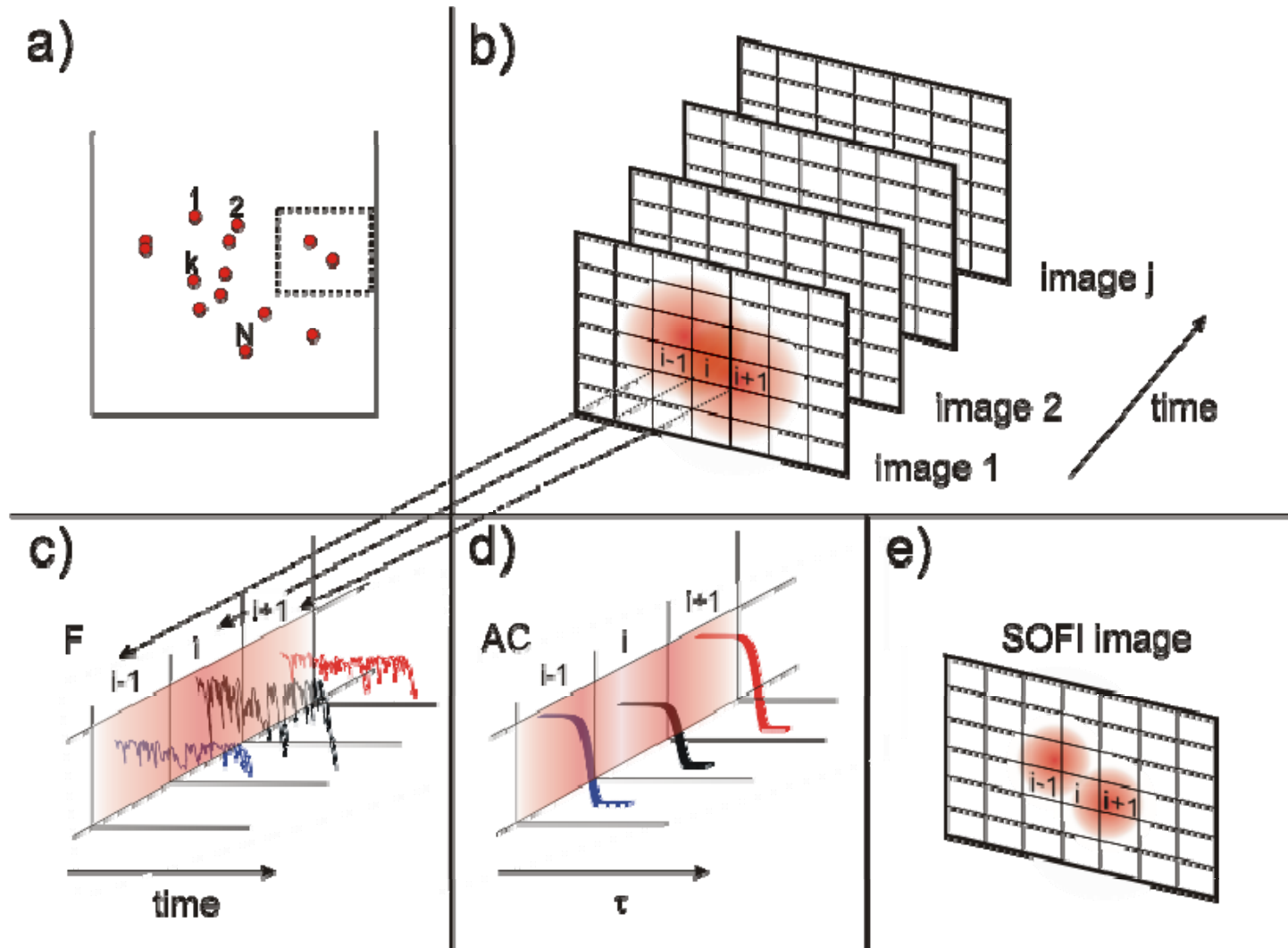
$$G_2(\mathbf{r}, \tau) = \langle \delta F(\mathbf{r}, t + \tau) \cdot \delta F(\mathbf{r}, t) \rangle_t =$$
$$= \sum_{k,l}^{\text{all molecules}} U(\mathbf{r} - \mathbf{r}_k) U(\mathbf{r} - \mathbf{r}_l) \varepsilon_k \varepsilon_l \langle \delta f_k(t + \tau) \delta f_l(t) \rangle_t$$

emitters fluctuate independently:  
i.e. emitter k correlates only with itself

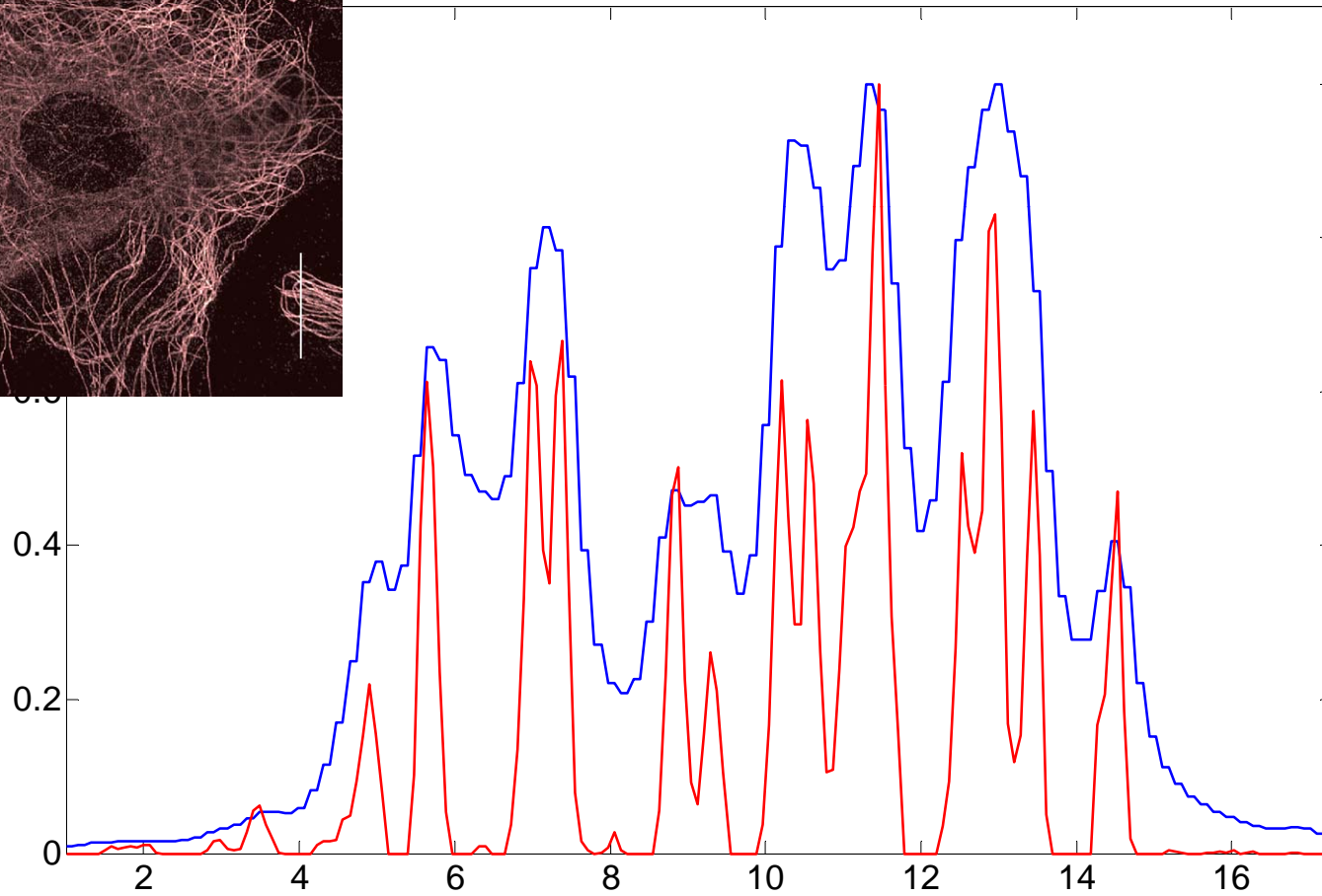
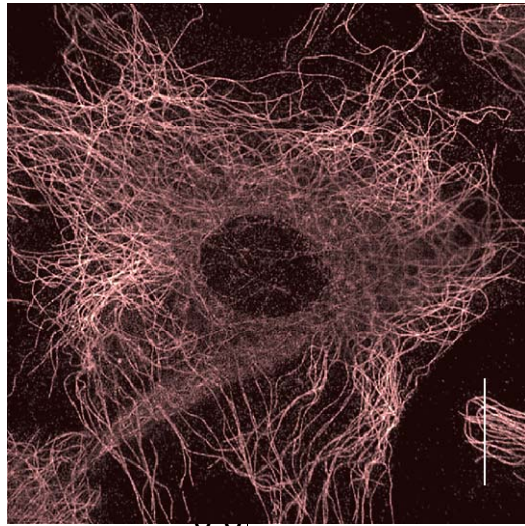
$$G_2(\mathbf{r}, \tau) = \sum_{k=1}^{\text{all molecules}} U^2(\mathbf{r} - \mathbf{r}_k) \varepsilon_k^2 \langle \delta f_k(t + \tau) \delta f_k(t) \rangle_t$$

square of PSF inside the  
sum over emitters

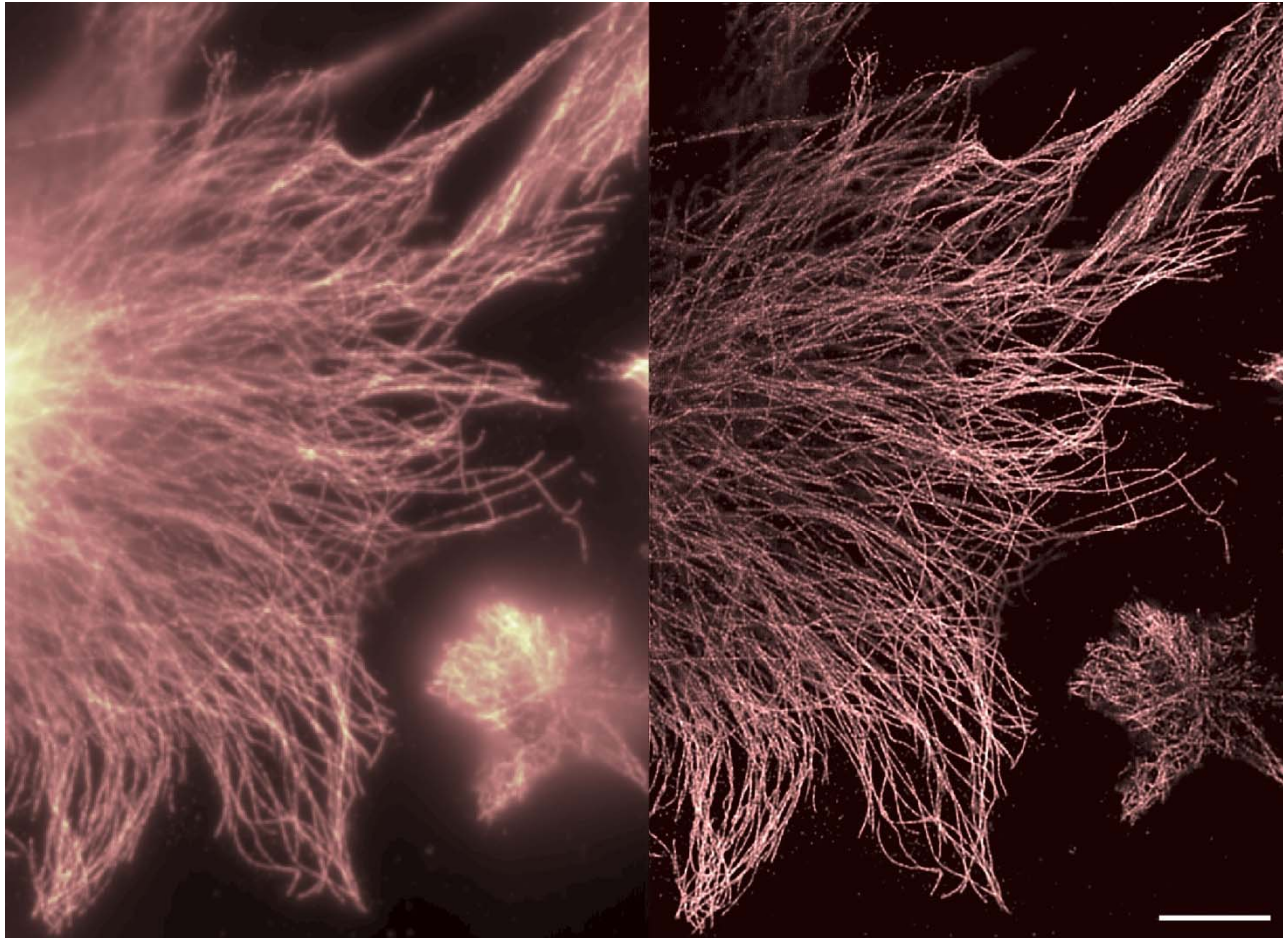
# SOFI scheme



# SOFI features Super-resolution in 3D

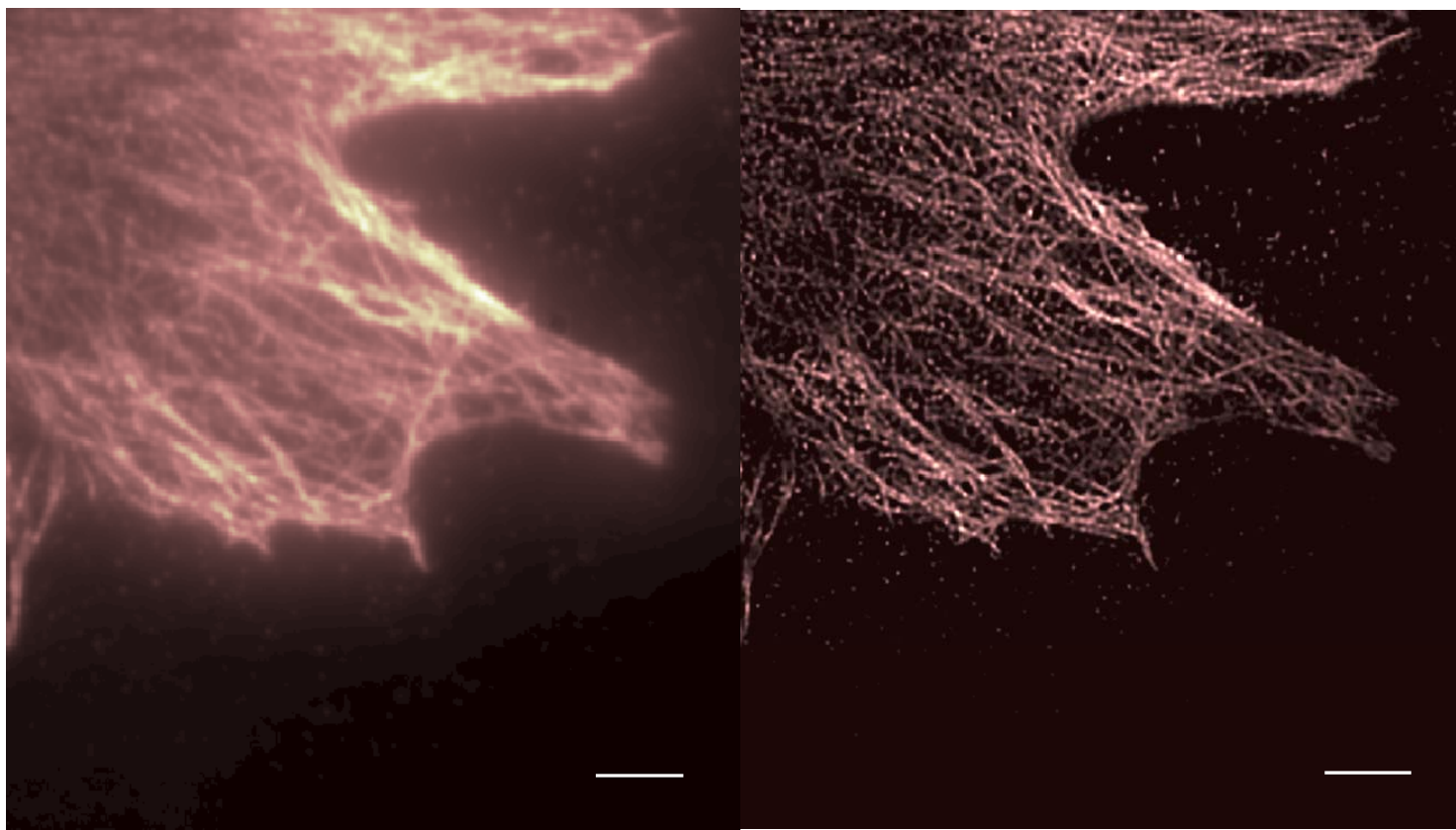


## SOFI features background-reduction and contrast-enhancement



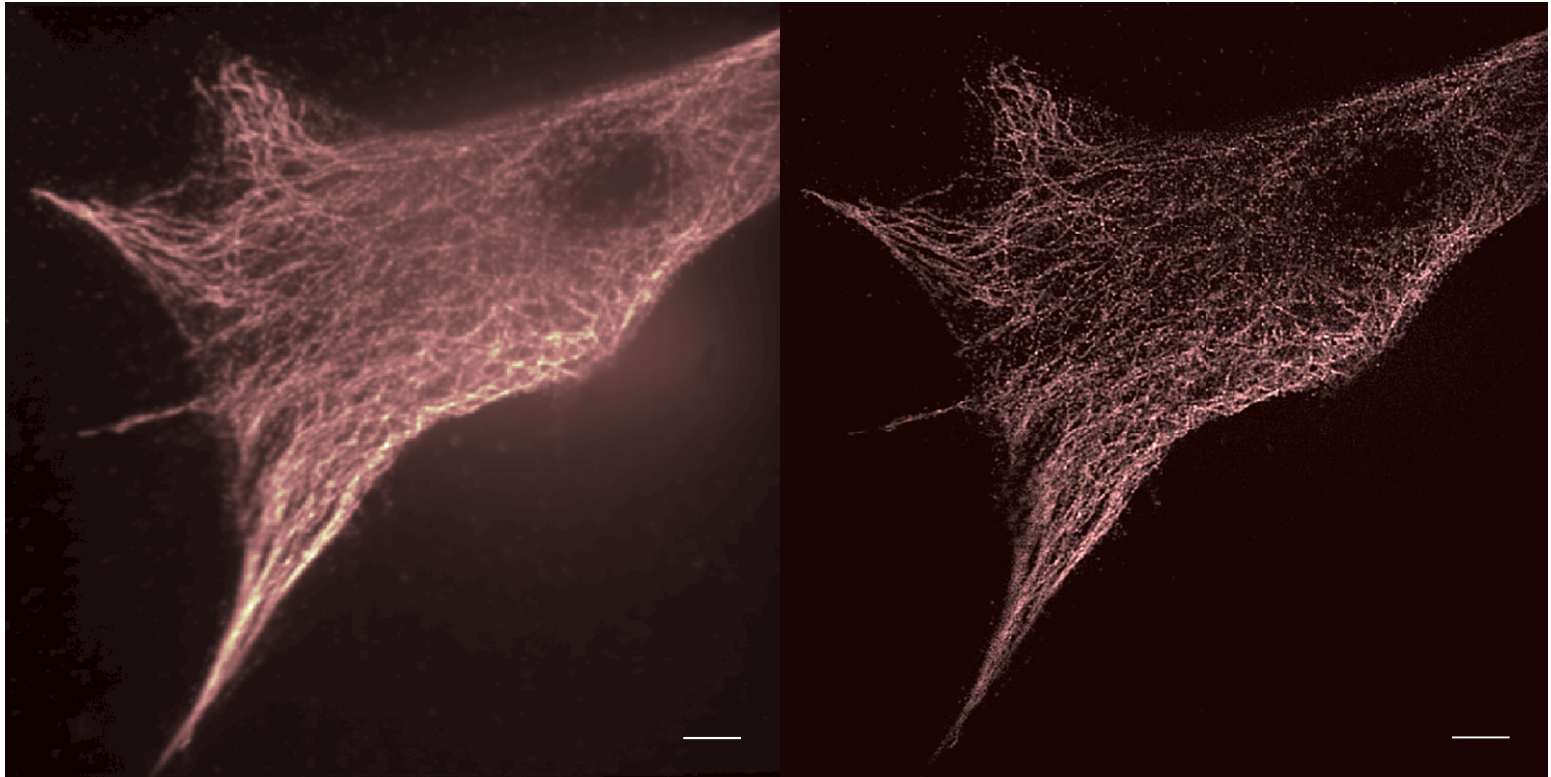
Scalebar: 10  $\mu\text{m}$

## SOFI on TIRF



Scalebar: 5  $\mu\text{m}$

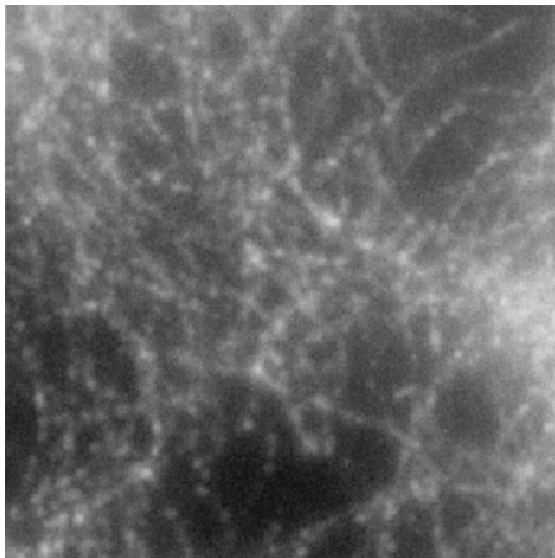
## SOFI on Spinning Disk Confocal



Scalebar: 5  $\mu\text{m}$

## SOFI with dyes

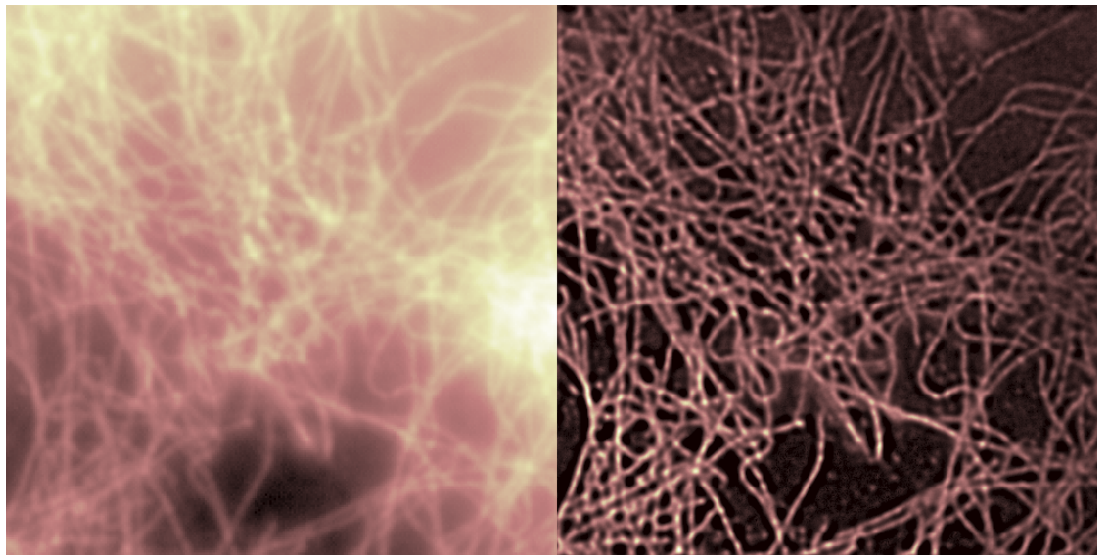
3T3 Fibroblasts, Alexa647 labeled tubulin network



In collaboration with Mike Heilemann, University of Bielfeld, Germany

## SOFI with dyes

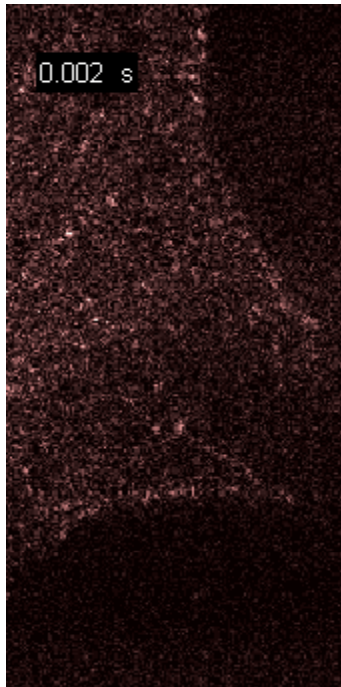
3T3 Fibroblasts, Alexa647 labeled tubulin network



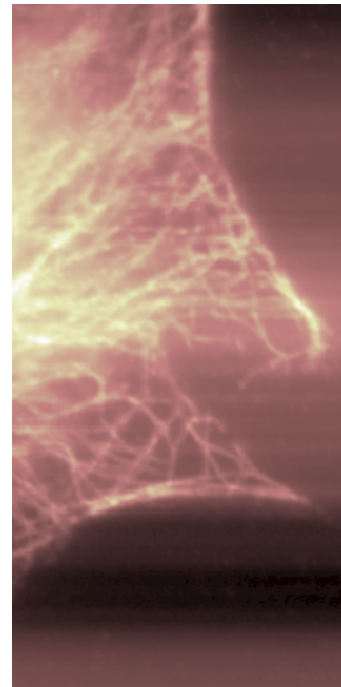
In collaboration with Mike Heilemann, University of Bielfeld, Germany

# Fast SOFI

SOFI image, movie taken at 1000 frames/s

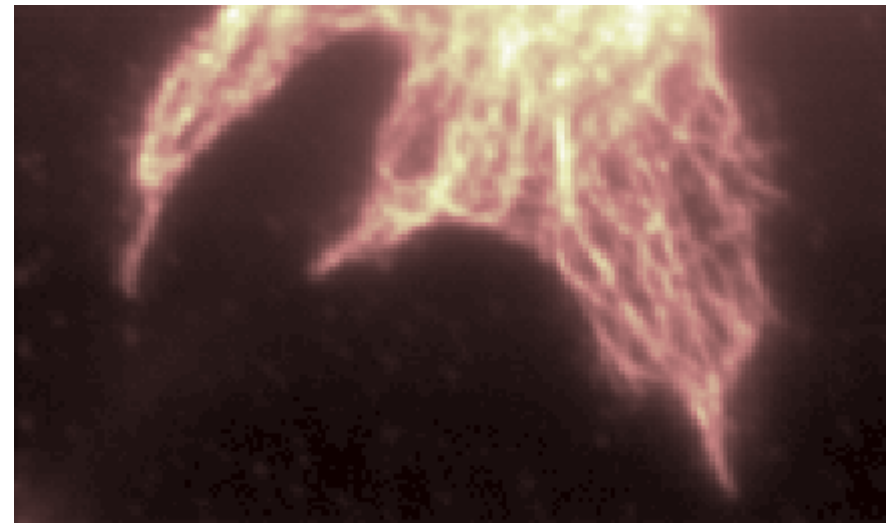
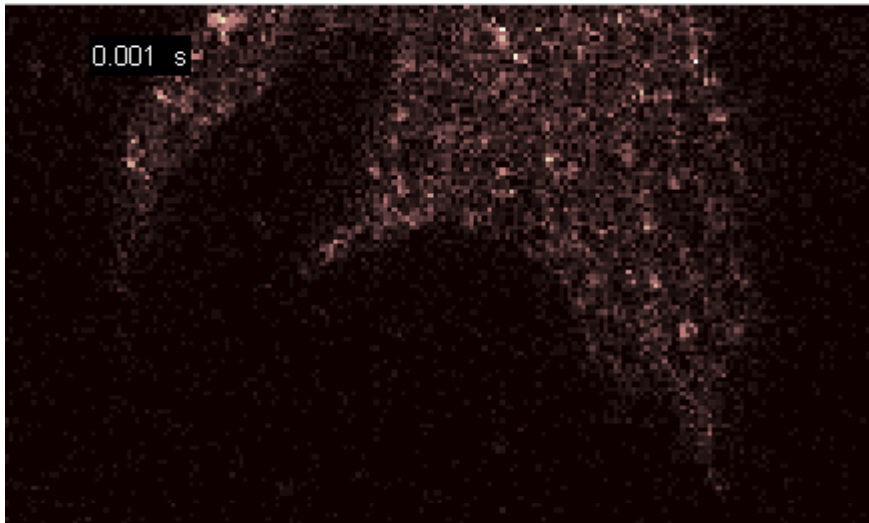


Original Image



## Fast SOFI

SOFI image, movie taken at 2000 frames/s



## Higher-order SOFI

Higher-order cumulants yield higher resolution

$n$ -th order correlation function:

$$G_n(\mathbf{r}, \tau) = \left\langle \delta F(\mathbf{r}, t) \cdot \delta F(\mathbf{r}, t + \tau_1) \cdot \dots \cdot \delta F(\mathbf{r}, t + \tau_{n-1}) \right\rangle_t$$

$n$ -th order cumulant:

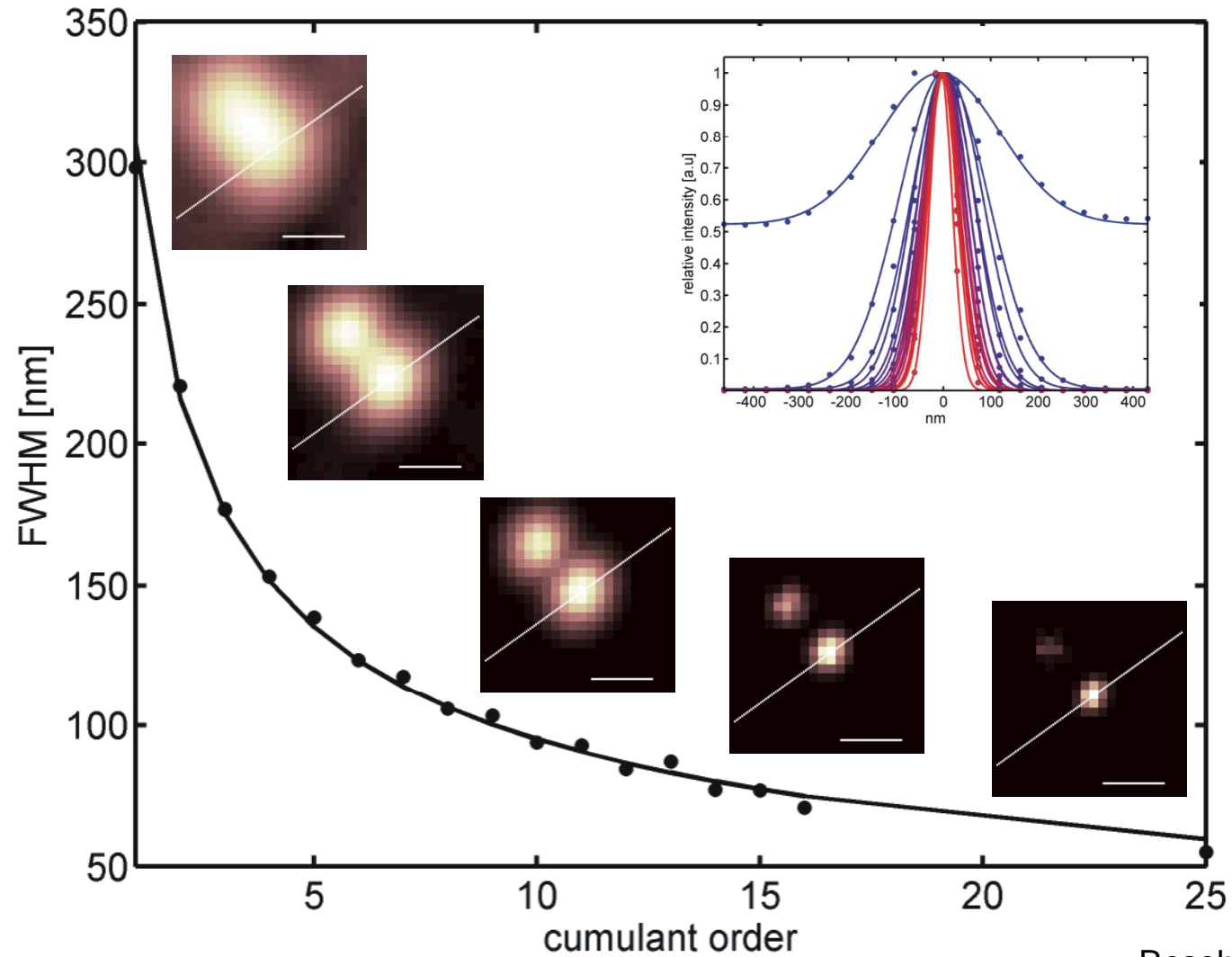
$$C_n(\mathbf{r}, \tau_1, \dots, \tau_{n-1}) = \text{function}(G_i) \propto \varepsilon^n U^n \quad i \leq n$$

$$C_2(\mathbf{r}, \tau_1) = G_2(\mathbf{r}, \tau_1)$$

$$C_3(\mathbf{r}, \tau_1, \tau_2) = G_3(\mathbf{r}, \tau_1, \tau_2)$$

$$C_4(\mathbf{r}, \tau_1, \tau_2, \tau_3) = G_4(\mathbf{r}, \tau_1, \tau_2, \tau_3) - G_2(\mathbf{r}, \tau_1) \cdot G_2(\mathbf{r}, \tau_3) - G_2(\mathbf{r}, \tau_1 + \tau_2) \cdot G_2(\mathbf{r}, \tau_2 + \tau_3) - G_2(\mathbf{r}, \tau_1 + \tau_2 + \tau_3) \cdot G_2(\mathbf{r}, \tau_2)$$

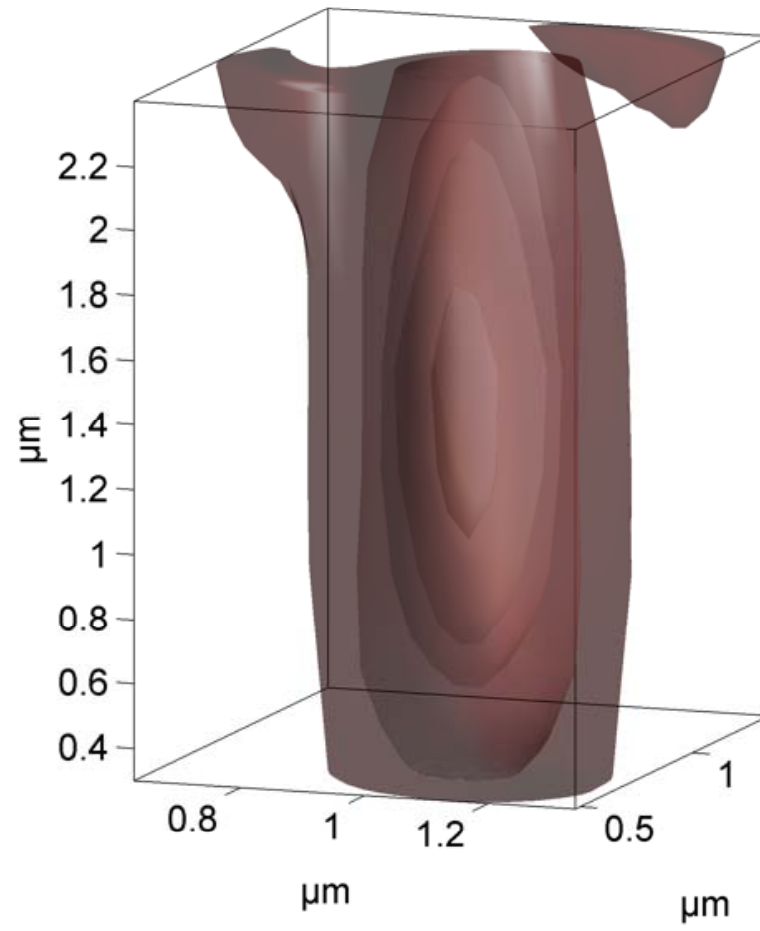
# HIGHER-ORDER SOFI



Scalebar: 250 nm

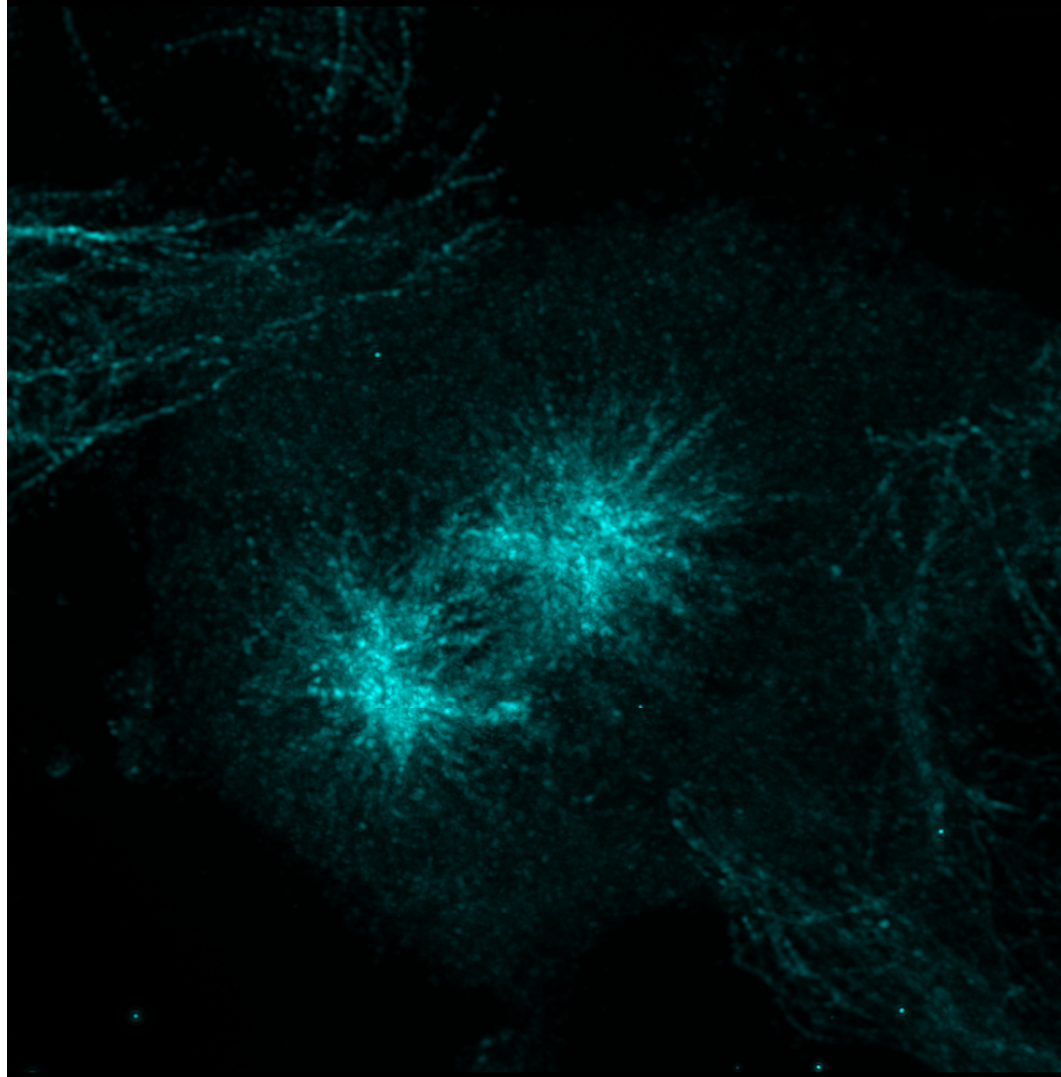
Resolution enhancement  
5x = 60 nm

# 3D-SOFI



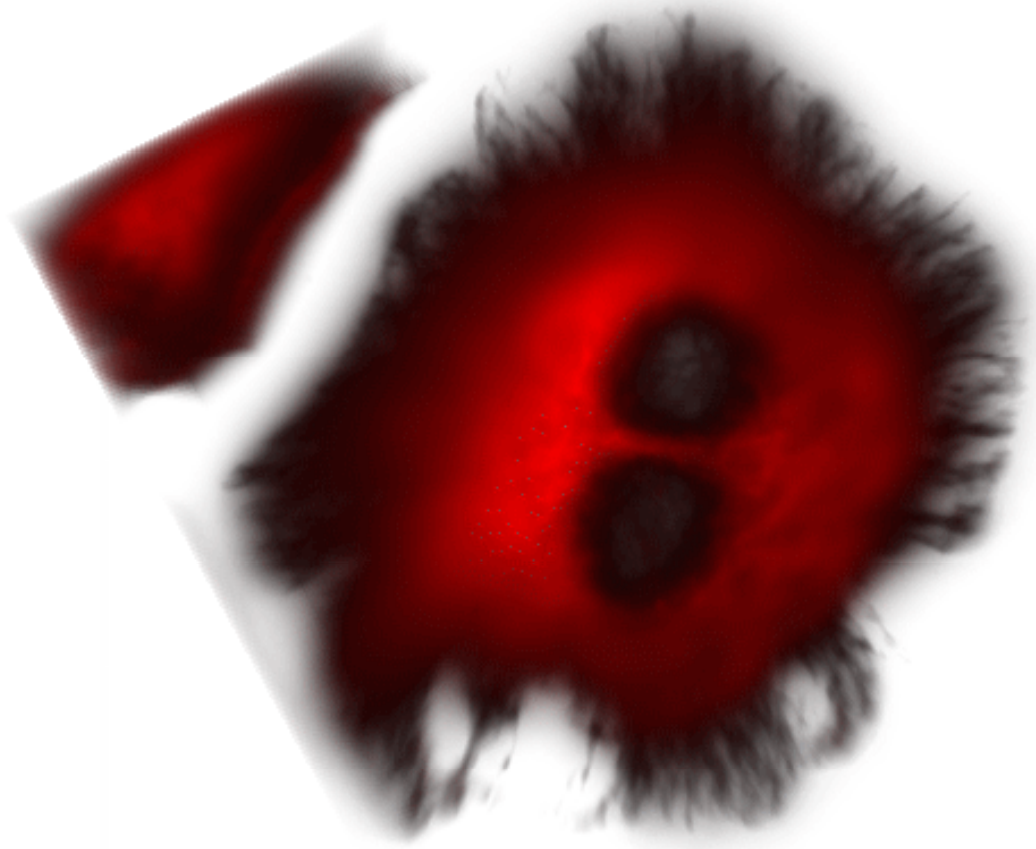
## 3D-SOFI

3D projection of 'wide-field' SOFI sections of QDs labeled  $\mu$ -tubules in HeLa cell



## 3D-SOFI

Fitst: 3D-rendering – of original data set



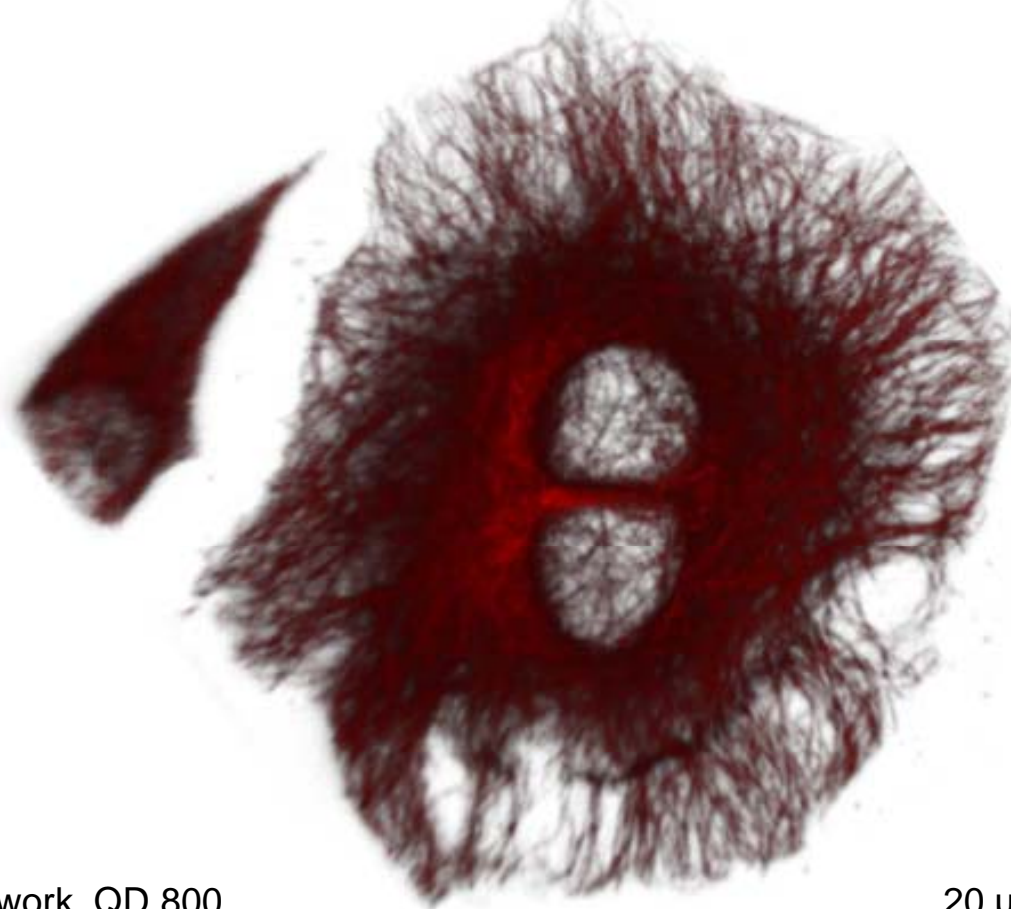
HeLa cell, tubulin network, QD 800,  
32 stacks, 6.4  $\mu\text{m}$  ( $\Delta z = 200\text{nm}$ ),  
2000 frames / stack,  
170 nm/pixel  
(Displaying at stretched z-aspect-ratio)

20  $\mu\text{m}$



## 3D-SOFI

3D projection of 'wide-field' SOFI sections of QDs labeled  $\mu$ -tubules in HeLa cell



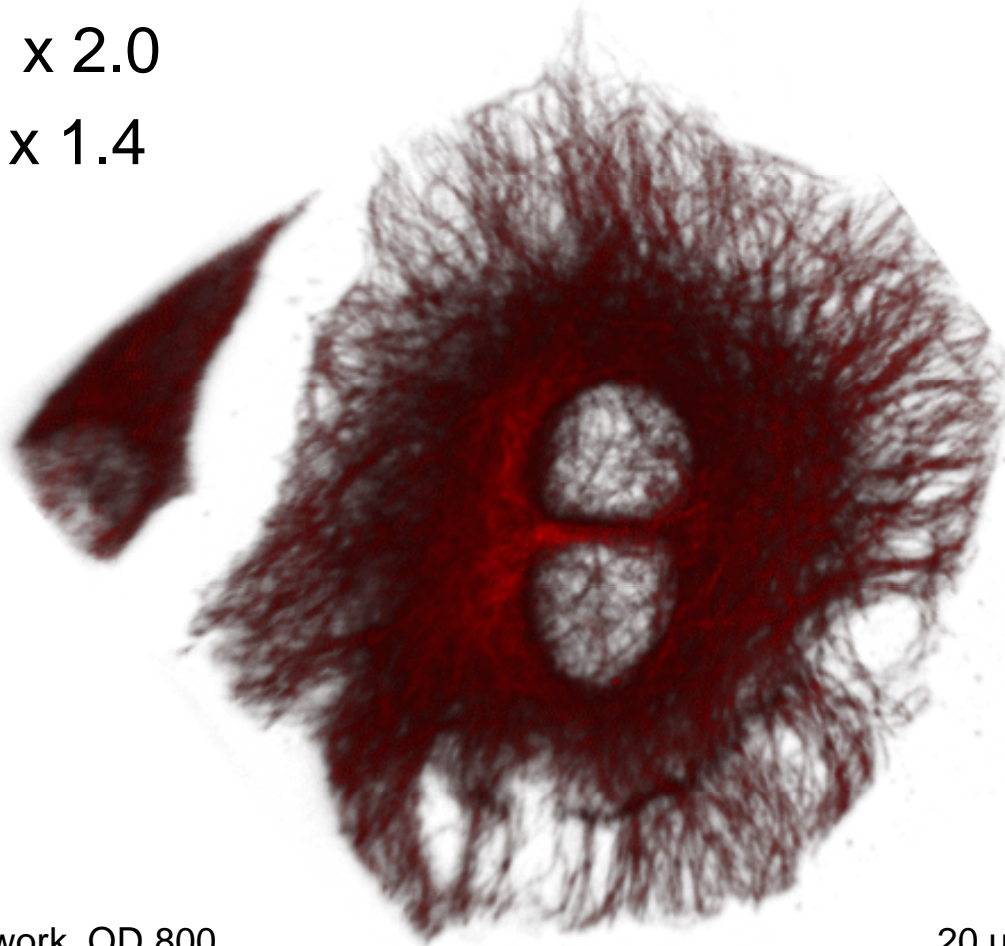
HeLa cell, tubulin network, QD 800,  
32 stacks, 6.4  $\mu\text{m}$  ( $\Delta z = 200\text{nm}$ ),  
2000 frames / stack,  
170 nm/pixel  
(Displaying at stretched z-aspect-ratio)

20  $\mu\text{m}$

## 3D-SOFI

Resolution enhancement:

- along x,y: x 2.0
- along z: x 1.4



20  $\mu\text{m}$

HeLa cell, tubulin network, QD 800,  
32 stacks, 6.4  $\mu\text{m}$  ( $\Delta z = 200\text{nm}$ ),  
2000 frames / stack,  
170 nm/pixel  
(Displaying at stretched z-aspect-ratio)

## Cross-Correlation Math

The equations for SOFI cross-correlations are very similar to the autocorrelations, with a few important differences:

$$\begin{aligned} \text{XC}_2(\mathbf{r}_1, \mathbf{r}_2, \tau) &= \langle \delta F(r_1, t + \tau) \cdot \delta F(r_2, t) \rangle_t \\ &= \sum_{k,l} U(\mathbf{r}_k - \mathbf{r}_1) U(\mathbf{r}_l - \mathbf{r}_2) \varepsilon_k \varepsilon_l \langle \delta f_k(t + \tau) \cdot \delta f_l(t) \rangle_t \end{aligned}$$

We still have a PSF<sup>2</sup> term inside the sum

$$\text{XC}_2(\mathbf{r}_1, \mathbf{r}_2, \tau) = U^2 \left( \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \right) \cdot \sum_k U^2 \left( \mathbf{r}_k - \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \cdot \varepsilon_k^2 \cdot \langle \delta f_k(t + \tau) \cdot \delta f_k(t) \rangle_t$$

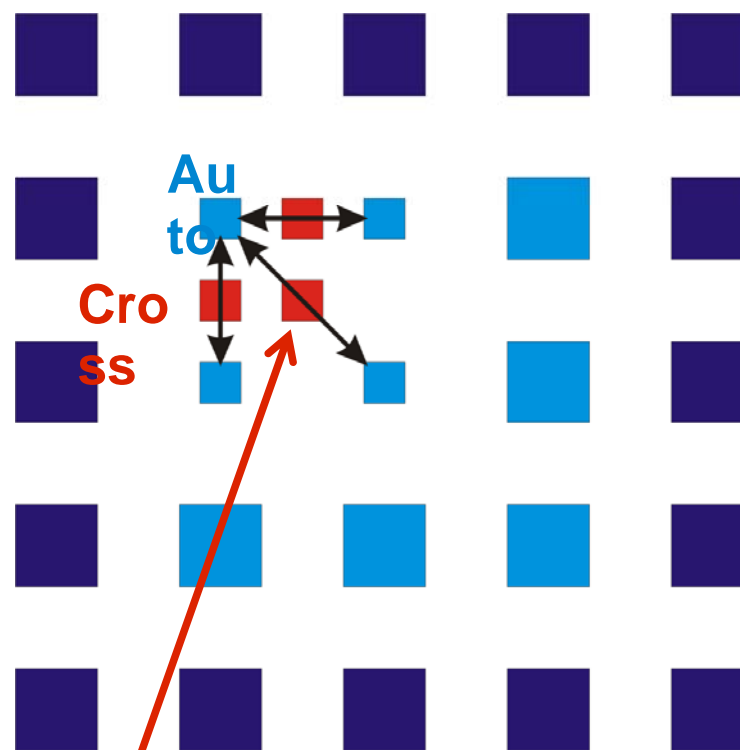
But now we have a **weighting term** out in front. This depends on the PSF and the distance between the two pixels.

We also have a new **“position”** term for the center of the resulting squared PSF!

It's the average of the two pixels!

# “Super-pixels”

- Each cross-correlation between neighboring pixels produces a new pixel with an effective “intermediate” location between them.
- When the autocorrelation and cross-correlations are combined, this results in 4 times the SOFI image pixels as the original image.
- This means we get new pixels for “free”. It gives us pixels with the resolution information from between two camera pixels.

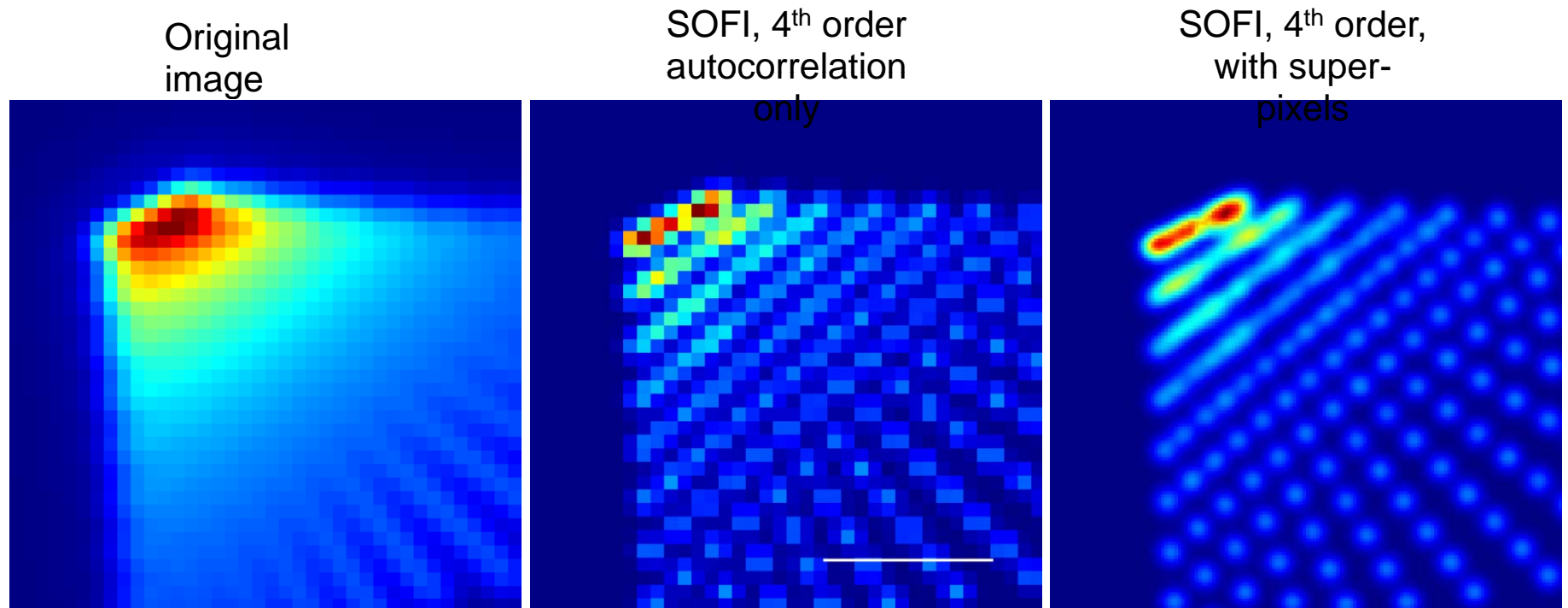


$$XC_2(\mathbf{r}_1, \mathbf{r}_2, \tau) = U^2 \left( \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \right) \cdot \sum_k U^2 \left( \mathbf{r}_k + \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \cdot \varepsilon_k^2 \cdot \langle \delta f_k(t + \tau) \cdot \delta f_k(t) \rangle_t$$

*Dertinger, et al., Optics Express 18(18), 18875-85 (2010).*

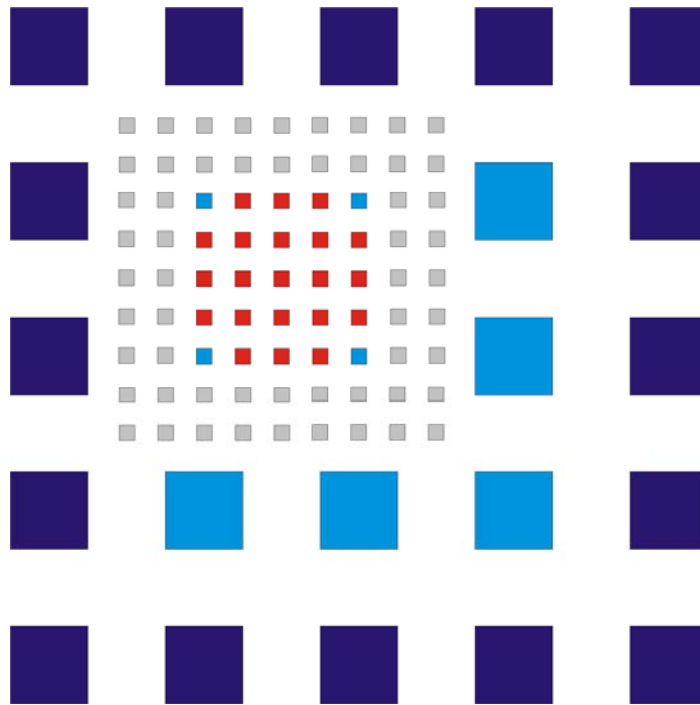
# “Super-pixels” Show True Resolution

- Here is a simulation which shows that the cross-correlated pixels truly do show “intermediate” pixel information which is not present in the original pixels.



# “Super-pixels”, higher orders

- These super-pixels scale along as we go to higher cumulant orders.
- The fourth order yields 4x4 times as many pixels as the original image.



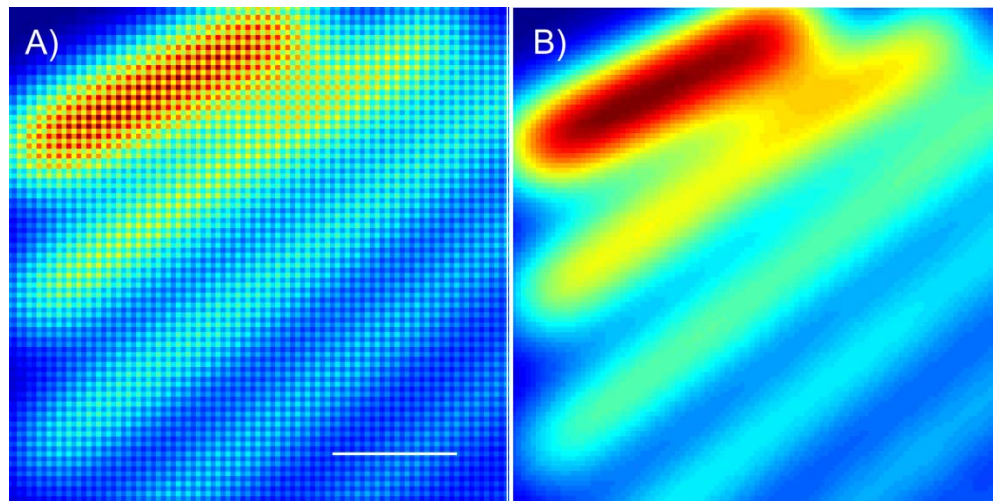
- This means that we never need to lose field of view as we gain resolution with SOFI!
- (Rule of thumb: Always choose a pixel size around half the PSF.)

# Weighting the Cross-Correlation

- Remember that “weighting term” in the cross-correlation?

$$XC_2(\mathbf{r}_1, \mathbf{r}_2, \tau) = U^2 \left( \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \right) \cdot \sum_k U^2 \left( \mathbf{r}_k - \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \cdot \varepsilon_k^2 \cdot \langle \delta f_k(t + \tau) \cdot \delta f_k(t) \rangle_t$$

- This makes all the values of the cross-correlation pixels “less” than their autocorrelation neighbors!

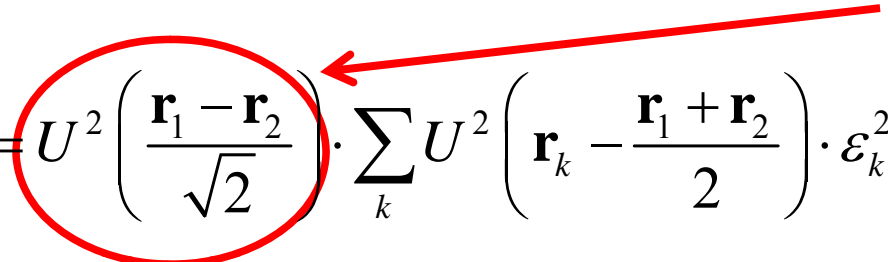


← Minimize the variance of all the pixels.

- So we can do a fit of the weighting coefficients to turn image (A) into the corrected image (B).

# Extracting the PSF Parameters

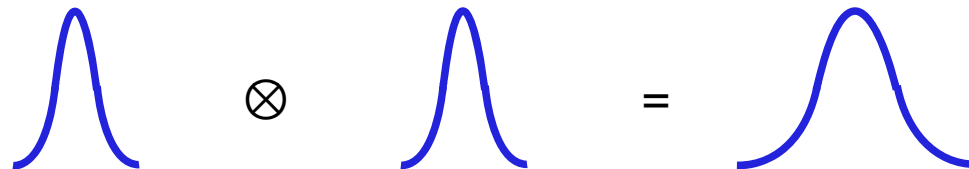
- As soon as we've done a fit of the weighting coefficients for a given sample, then we have found an experimental value of the this term.

$$XC_2(\mathbf{r}_1, \mathbf{r}_2, \tau) = U^2\left(\frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}\right) \cdot \sum_k U^2\left(\mathbf{r}_k - \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) \cdot \varepsilon_k^2 \cdot \langle \delta f_k(t + \tau) \cdot \delta f_k(t) \rangle_t$$


- And, we know  $r_1 - r_2$ , because that's simply the distance between two pixels. Call it "1" for neighbors.
- Since  $U$  is the PSF function, that means that by our fit, we have experimentally measured the width of the PSF in units of pixels!
- We get the PSF width for free, with no extra measurements, as a byproduct of computing the cross-correlation components.
- So let's see what we can do with that PSF information...

# The Optical Transfer Function under SOFI

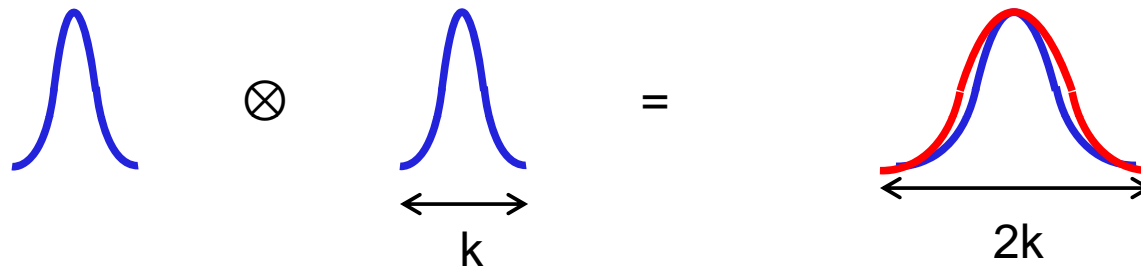
- The Fourier transform of a Gaussian PSF is a Gaussian in frequency space.
- Squaring the PSF convolves the frequency space Gaussians:


$$\text{Gaussian} \otimes \text{Gaussian} = \text{Wider Gaussian}$$

- The new Gaussian is only  $\sqrt{2}$  wider by the FWHM.
- However, by the math of convolution, the “support”, or information content, of this optical transfer function (OTF) goes **twice** as far out as the original.

# Improving Resolution with the “OTF”

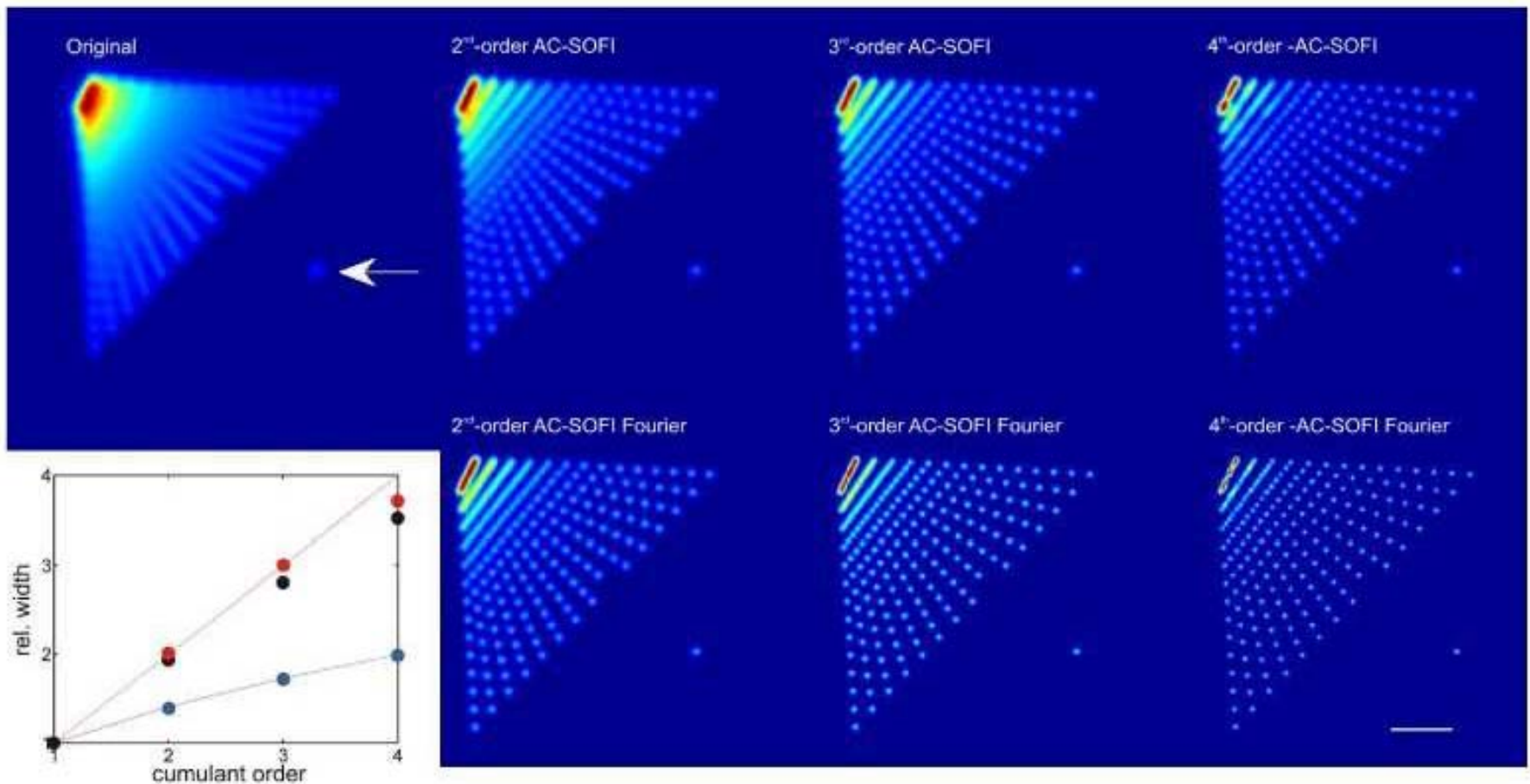
- Since the support of the OTF goes twice as far out, and since we know the precise PSF dimensions from the cross-correlation weighting...



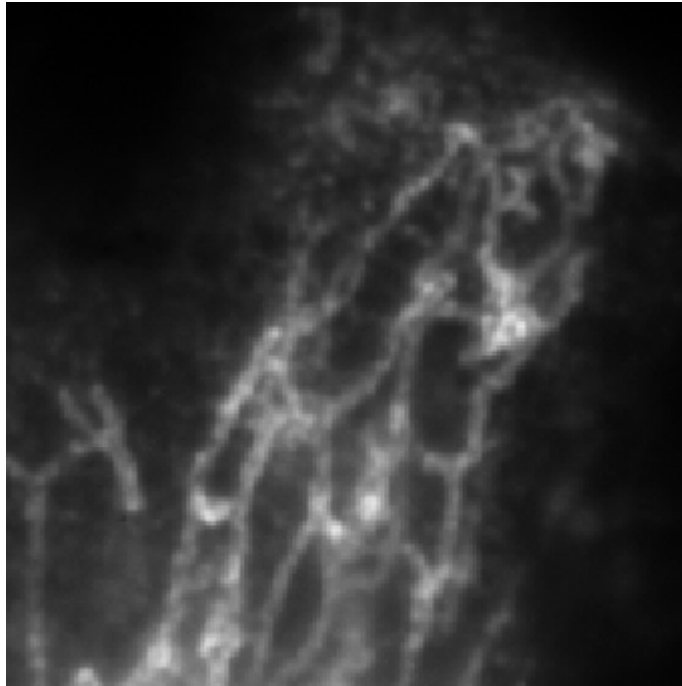
- If we simply reshape the frequency space Gaussian by multiplying it back to the full amplitude, we will recover the full information which is actually there, giving us twice the spatial frequency coverage.
- When this goes back into real space, it means 2X resolution from the second order rather than the square root of two!

# Superresolution Which Scales Linearly

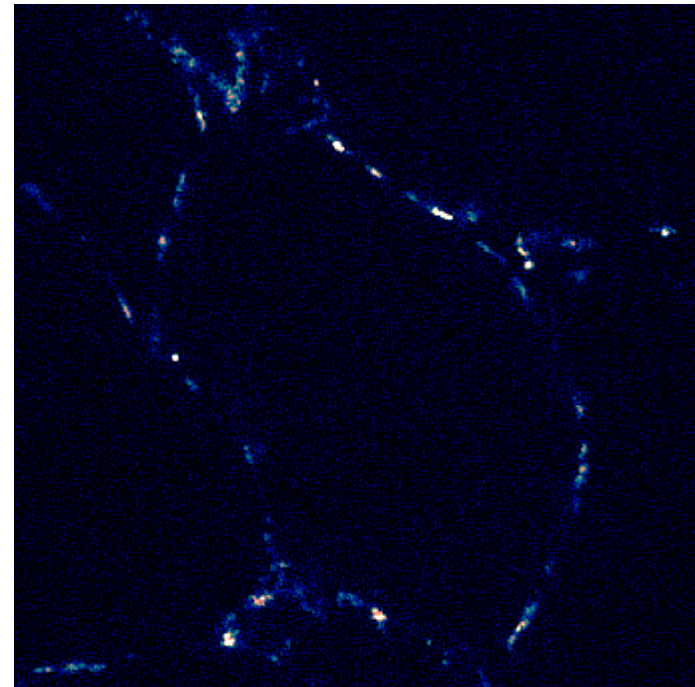
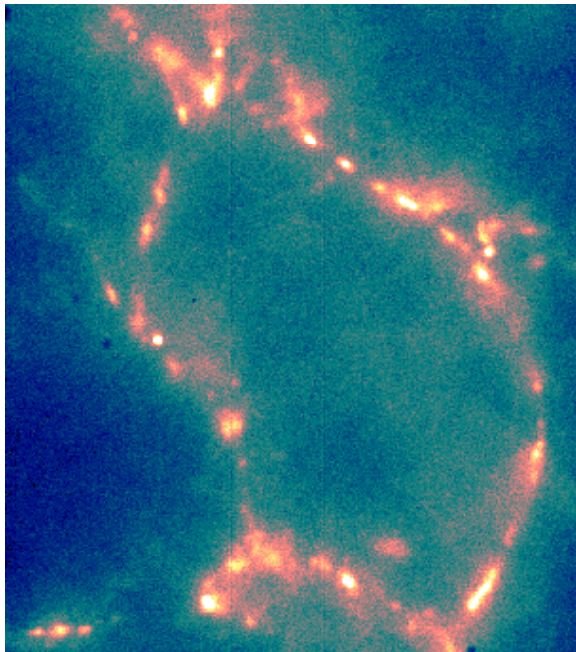
- When the full OTF correction is performed, as in this simulation, the resolution scales linearly with cumulant order! 4<sup>th</sup> order = 4X.



## **SOFI on tubulin network in live cells**



## SOFI on EGFR



MCF7\_EGFR\_800\_\_3, 500 frames blocklength 50, 70nm/pixel

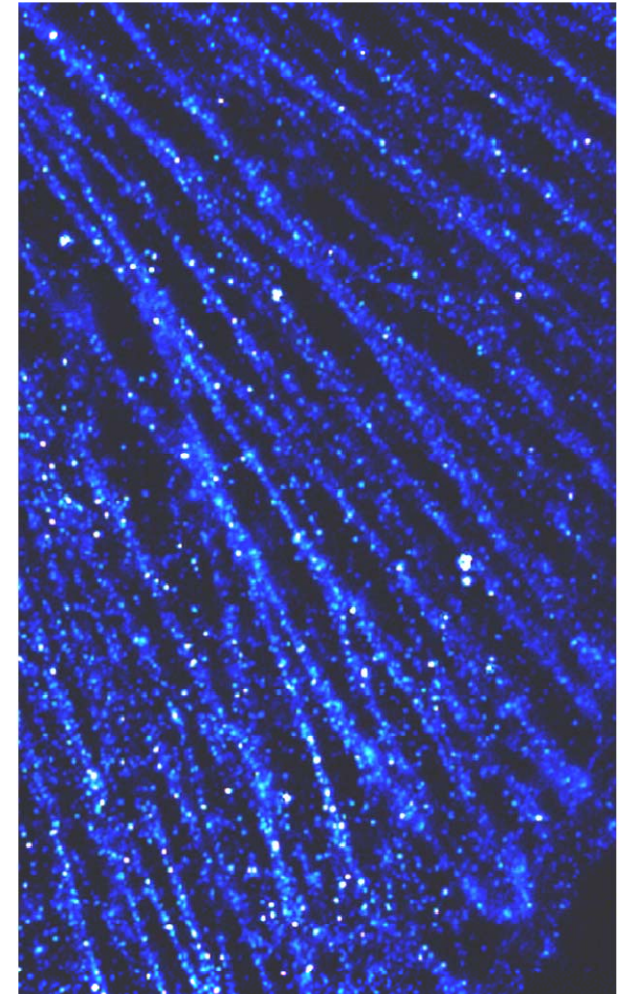
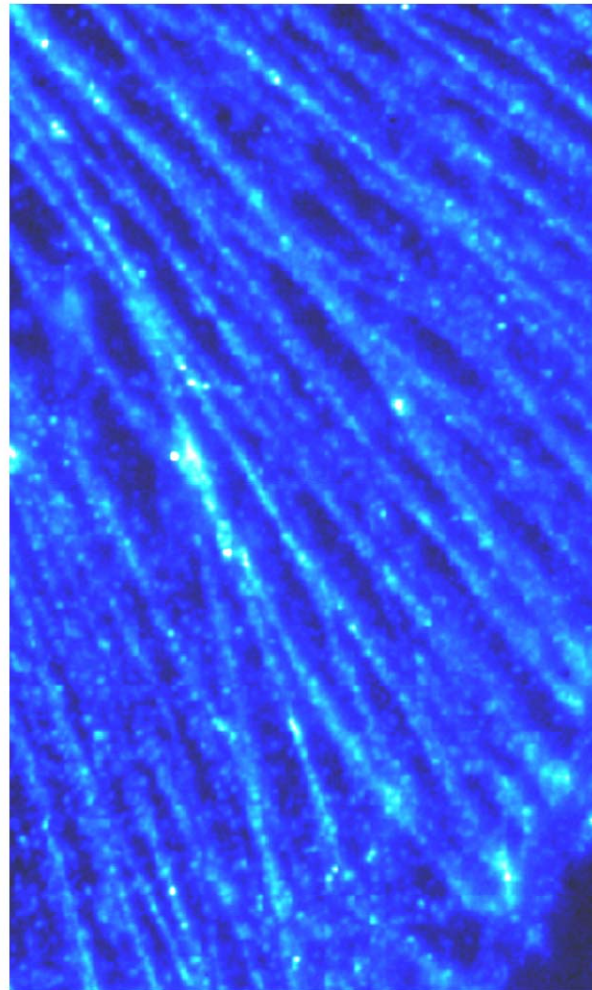
# SOFI on FAPs

FAP:  
Fluorogen Activating Protein

Single-chain antibodies evolved to  
bind malachite green dye.

Acquisition time: 200s  
TIRF microscope.  
150x magnification

In collaboration with Marcel  
Bruchez at Carnegie Mellon  
University



## SOFI summary

- Implementation of SOFI is simple
- It is inherently 3D
- It is background-free
- It can be fast
- It works on all blinking probes (i.e. almost all probes)
- It works on all types of microscopes
- Resolution is not limited to pixel size

# Acknowledgements

**Thomas Dertinger**

**Ryan Colyer**

**Robert Vogel**

**Gopal Iyer**

**Jörg Enderlein (University Göttingen)**

Dertinger et al., PNAS 2009; Optics Express 2010; Angew Chemie 2010

# **A Novel Detector**

**Xavier Michalet**

**Ryan Colyer**

**Oswald Siegmund**

**Anton Tremsin**

**Paul Vallergera**

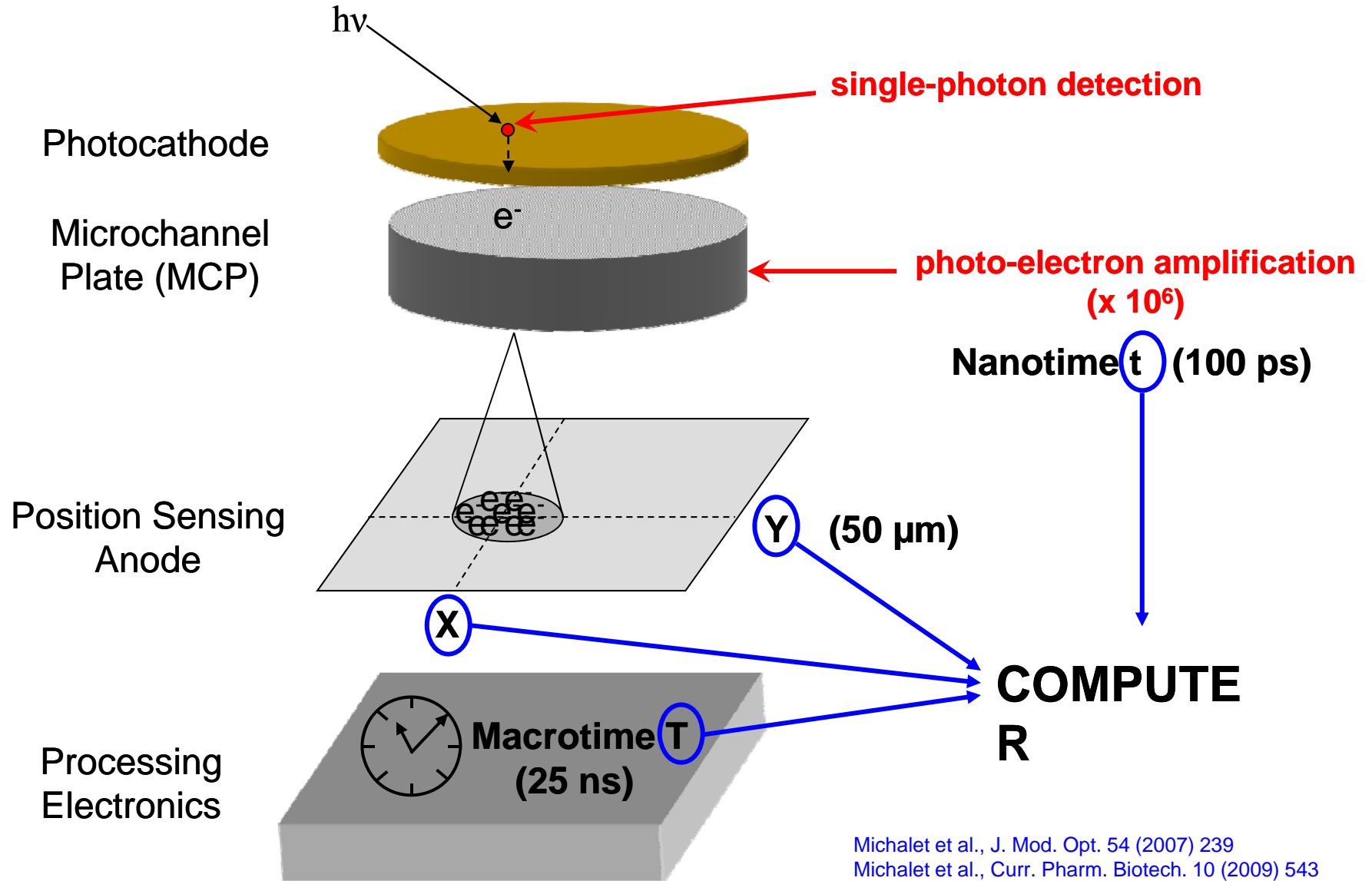
**Paul Hinks (Photonis)**

# A new single-molecule photon-counting camera

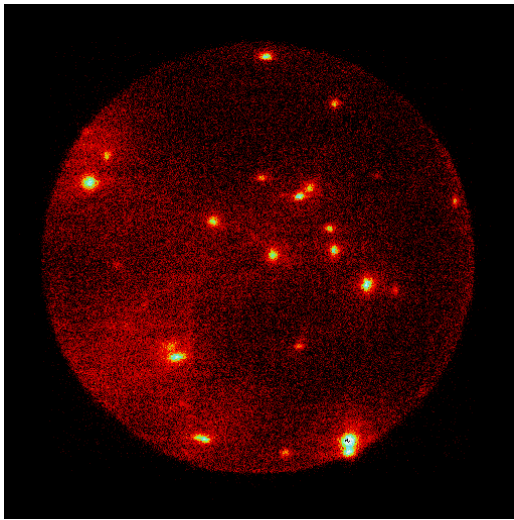
+

# The H33D (“heed”) Detector

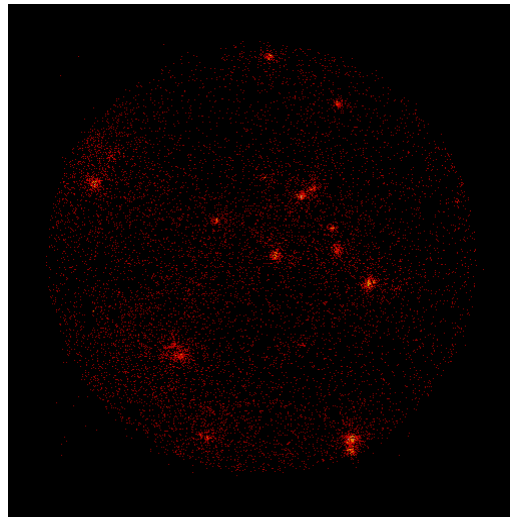
*High spatial, High temporal resolution, High throughput, 2+1D detector*



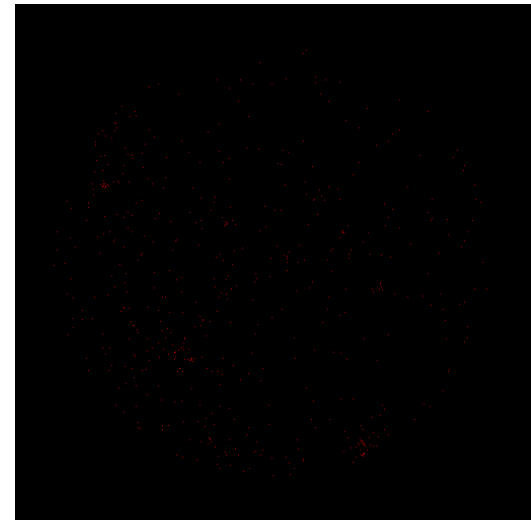
## The H33D in action...



10 fps



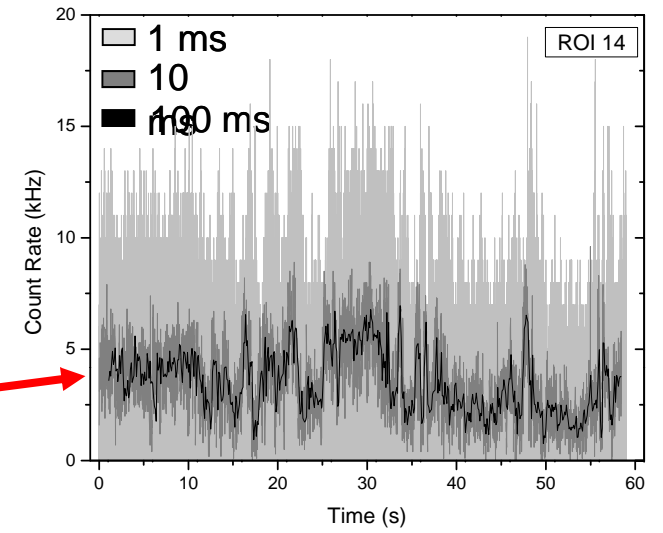
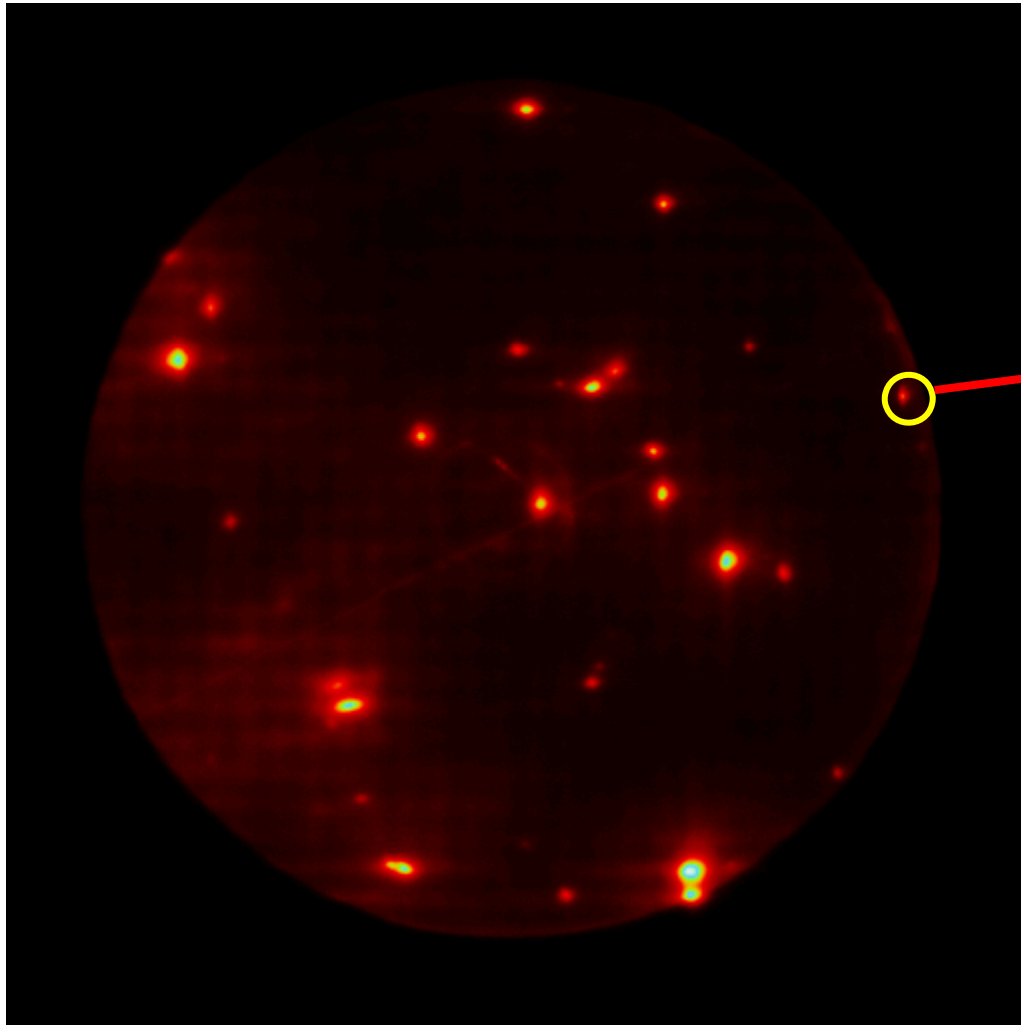
100 fps



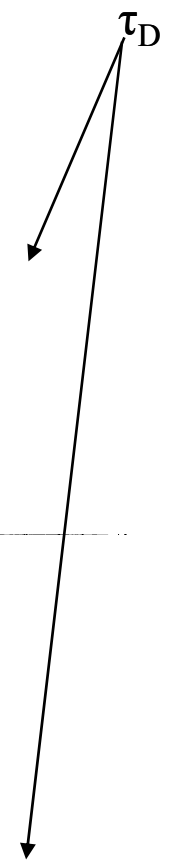
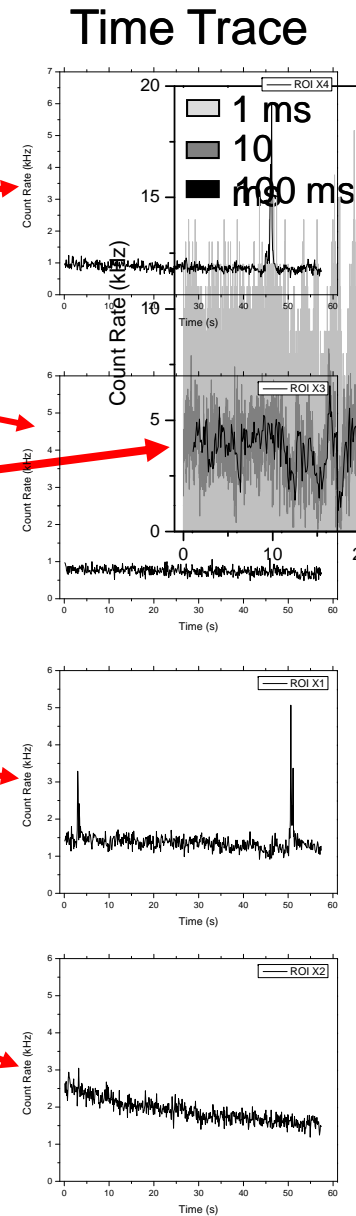
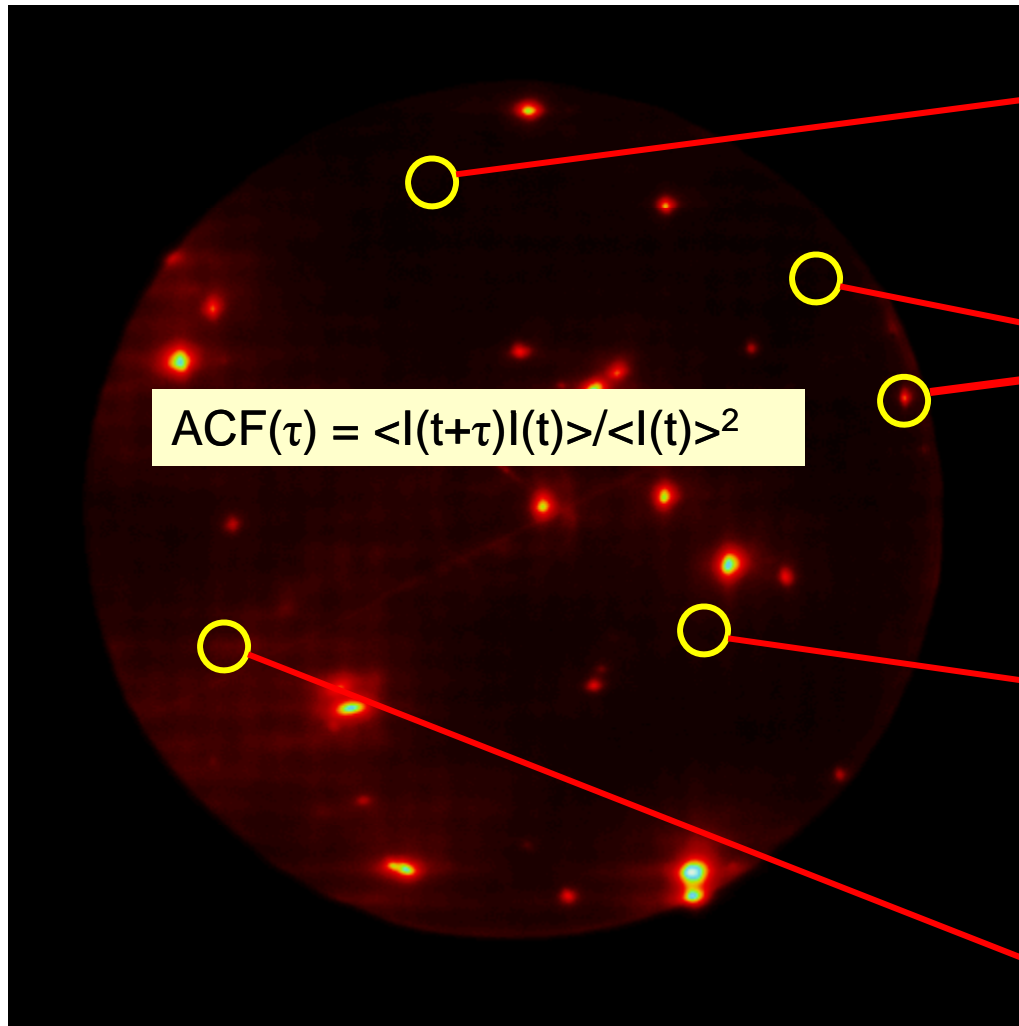
1000 fps

**Xavier Michalet, Anton Tremsin, Paul Vallerger, Oswald Siegmund, Photonis**

# The H33D's detector timing resolution



# Beyond tracking and beyond single-molecule...



# **A Novel Detector**

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**Oswald Siegmund**

**Anton Tremsin**

**Paul Vallergera**

**Paul Hinks (Photonis)**